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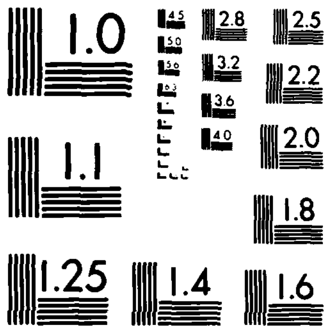
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REPORT

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The structure of a weak normal shock wave is investigated analytically for the one-dimensional flow of a fluid, containing gas bubbles. The relative motion of the gas bubbles and fluid are allowed for, and it is assumed that the gas bubbles follow a Stokes-drag law. It is shown that for large values of time the gas bubbles behave isothermally and the shock wave approaches a steady state. For small values of time the gas bubbles behave isentropically and move faster than the fluid.

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The structure of a weak normal shock wave is investigated analytically for the one-dimensional flow of a fluid, containing gas bubbles. The relative motion of the gas bubbles and fluid are allowed for, and it is assumed that the gas bubbles follow a Stokes-drag law. It is shown that for large values of time the gas bubbles behave isothermally and the shock wave approaches a steady state. For small values of time the gas bubbles behave isentropically and move faster than the fluid.

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C O N T E N T S

WEAK SHOCK WAVES PROPAGATING IN FLUID-GAS MIXTURES

1. INTRODUCTION

Suspensions of solid, liquid or gaseous particles in an ambient fluid are common both in nature and industrial processes. Interesting military examples are fuel-air explosives, aerated liquid explosives, foams (Dewey 1981, 1983) and bubble curtains (Mader 1982). There is some interest therefore to understand the basic physical mechanisms that occur when a liquid-gas mixture is shock loaded.

It is well known that the presence of gas bubbles in an homogeneous explosive significantly increases the sensitivity of the explosive, Campbell et.al. (1961), Evans et. al. (1962) and Mader (1965). Their results show that when bubbles in the explosive are heated significantly above the temperature of the liquid due to impact with the walls of the bubble, hot spots form in the explosive and detonation can result.

The attenuation of shock waves by foams is of particular interest where protection from blast waves is needed. Liquid foams have physical characteristics such as low velocity of propagation of sound, high diffusion losses, and high heat capacity losses making them suitable as blast suppressants.

Shock waves in fluids containing gas bubbles have been considered to some extent by Crespo (1969) and Campbell and Pitcher (1957) and the references contained therein. Crespo considers the propagation of infinitesimal sound waves in a fluid containing gas bubbles and the structure of steady shock waves. Campbell and Pitcher have derived the Hugoniot relations (i.e. relations that express conservation of mass and momentum across the shock) for a normal steady shock in a fluid containing gas bubbles, neglecting temperature rises across the shock. They have also reported experimental measurements of shock waves in bubble-liquid mixtures using a small gas-liquid shock tube.

In the following paper a simple two-phase flow model is developed for the shock transition region of a weak shock wave propagating into a fluid containing spherical gas bubbles whose size is small compared to the shock transition zone. The shock wave is generated by the impulsive motion of a piston. Assumptions made in deriving an analytical solution are similar to those of Moran and Shen (1966), Campbell and Pitcher (1977), Crespo (1969) and Van Wijngaarden (1970). They are (i) the fluid is incompressible and the gas follows the perfect-gas law, (ii) boundary layer effects can be neglected, (iii) the only viscous effects are due to the gas bubble drag and the only heat transfer is that between the surrounding liquid and gas bubbles, (iv) the thermal motion of the gas bubbles is negligible and so does not contribute to the pressure and (v) the shock wave moves with the velocity of the piston.

The shock structure is investigated in the context of the linear theory i.e., the near field solution. The methodology is to expand all dependent state variables (which can be collectively designated ψ) in the Navier-Stokes equations describing the flow, as $\psi = \psi_o + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \ldots$, and equate terms of order $O(\varepsilon)$ or higher to zero. The procedure to be followed in solving this set of coupled linear partial differential equations with assigned boundary and initial conditions is to use integral transform methods. The solution gives an approximation to the near-field fluid flow.

2. BASIC EQUATIONS

The shock transition for a weak shock wave propagating through an inviscid and incompressible fluid containing gas bubbles is shown in figure 1.

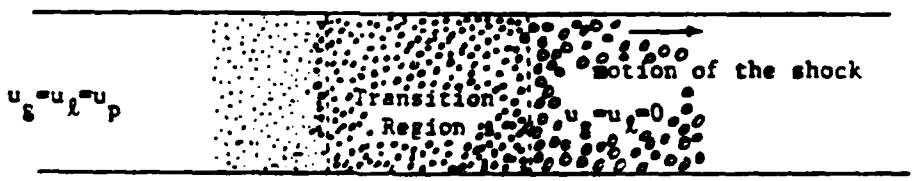

Figure 1. Shock wave propagating through a fluid-gas mixture.

The conditions of the gas are described by its velocity (u_g) and by two state variables density (ρ_g) and temperature (T_g). The suffix ℓ or g to a symbol indicates that it refers to the liquid or gas component. Other state variables such as pressure (P_g) or the speed of sound may be introduced through appropriate thermodynamic relations such as an equation of state of the gas. Since the radius of the gas bubbles are assumed small compared to the thickness of the shock, the pressure of the gas (P_g) can be assumed equal to that of the liquid (P_ℓ). The state variables of the liquid are characterised by its velocity (u_ℓ), temperature (T_ℓ) and volume fraction of gas in the fluid (β).

In a cartesian coordinate system, the one-dimensional time dependent equations describing the conservation of mass, momentum and energy of the fluid-gas mixture are given as follows. Let the density of the mixture ρ_m be given in terms of the densities of the two components by: $\rho_m = (1-\beta)\rho_\ell + \beta\rho_g$. Assuming no mass exchange between the phases, the continuity equations expressed in terms of the volume fraction of gas β and volume fraction of liquid $(1-\beta)$ are:

$$\partial_t(1-\beta) + \partial_x(1-\beta)u_\ell = 0, \tag{2.1}$$

for the liquid and

$$\partial_t(\beta\rho_g) + \partial_x(\beta\rho_g u_g) = 0, \tag{2.2}$$

for the gas bubbles, where the density of the liquid ρ_ℓ in (2.1) has been cancelled out because the fluid is assumed incompressible.

The equation of momentum of the fluid-gas mixture is given by

$$\partial_x P_\ell = -\rho_\ell(1-\beta)\frac{Du_\ell}{Dt_{(\ell)}}, \tag{2.3}$$

where

$$\frac{D}{Dt_{(j)}} := \partial_t + u_{(j)}\partial_x.$$

is the differential following the motion of the fluid, the index j can be either g or ℓ for gas or liquid respectively.

The momentum equation for a volume of gas can be written following Crespo (1969) as

$$\partial_x P_g = \tfrac{1}{2}\rho_\ell\Gamma\frac{Du_{\ell g}}{Dt_{(g)}} + \frac{f}{V}, \tag{2.4a}$$

where $u_{\ell g} := u_\ell - u_g$, f is the internal force, V is the volume of a gas bubble, and Γ a function which depends on β. For spherical bubbles and β small, $\Gamma \approx 1$. The force on a gas bubble moving through the fluid has been considered by Van Wijngaarden (1970) and Levich (1962). For high Reynolds numbers the relative motion of the gas bubble can be determined from the irrotational inviscid flow around the bubble. Since the gas bubble is free to move there is no constraint on the tangential velocity of the fluid at the surface of the bubble. Hence no velocity boundary layer exists. Thus the frictional force acting on the bubble can be taken to approximate a Stokes law i.e., the drag force on a spherical bubble is proportional to the relative fluid velocity:

$$f = 6\pi r_g \mu_\ell u_{\ell g},$$

where μ_ℓ is the dynamic viscosity of the fluid and r_g the initial radius of a gas sphere. The term $\frac{f}{V}$ appearing in (2.4a) can now be expressed as follows:

$$\frac{\rho_g f}{m} = \frac{9}{2} u_{\ell g} \rho_\ell \nu_\ell \left(\frac{4\pi\rho_g}{3m}\right)^{2/3},$$

where ν_ℓ is the kinematic viscosity of the fluid. This equation represents the dissipative force per unit volume which in terms of the drag coefficient C_D can be written as

$$\frac{\rho_g}{m} f = \frac{3}{8} \frac{\rho_\ell}{r_g} u_{\ell g} |u_{\ell g}| C_D,$$

where

$$C_D = \frac{12\nu_\ell}{|u_{\ell g}|} \left(\frac{4\pi\rho_g}{3m}\right)^{1/3}.$$

In terms of the momentum relaxation time α_m:

$$\alpha_m := \frac{m}{6\pi r_g \mu_\ell} = \frac{2}{9} \frac{\rho_g r_g^2}{\mu_\ell},$$

the momentum equation of the gas can be expressed as

$$\partial_x P_g = \frac{1}{2} \rho_\ell \Gamma \frac{Du_{\ell g}}{Dt_{(g)}} + \rho_g \frac{u_{\ell g}}{\alpha_m}, \qquad (2.4b)$$

where the second term of (2.4b) represents a non-dissipative force per unit volume due to the mass of the bubble.

The equation of energy of the gas can be written as

$$\frac{DP_g}{Dt_{(g)}} - \frac{\gamma_g P_g}{\rho_g} \frac{D\rho_g}{Dt_{(g)}} = (\gamma_g - 1)Q, \qquad (2.5)$$

where Q is the heat transfer rate and γ_g the ratio of specific heats of the gas. Crespo (1969) gives Q as

$$Q = \frac{2\pi\rho_g}{m} r_g \sigma_\ell (T_o - T_g) Nu,$$

where σ_ℓ and T_o are the thermal conductivity and the initial temperature of the liquid respectively. Nu is the Nusselt number. For weak wave analysis we assume there is only pure heat conduction, thus Nu = 2 and

$$Q = 3\left(\frac{4\pi\rho_g}{3m}\right)^{2/3} \sigma_\ell (T_o - T_g) \; .$$

The final equation is the simple equation of state of the gas:

$$P_g = \rho_g R T_g. \tag{2.6}$$

Equations (2.1) through to (2.6) give six simultaneous equations in x and t for the fluid flow variables $\psi = (P_g, u_\ell, u_g, T_g, \rho_g, \beta)$. These must be solved for the spatial and temporal dependences. Where a shock wave propagates down a semi-infinite shock tube, the initial conditions are

$$u_\ell = u_g = 0; \quad x > 0,$$

$$P_\ell = P_o, \; \rho_g = \rho_o, \; T_\ell = T_g,$$

and the boundary conditions are

$$u_\ell = U_p, \; t > 0, \; x = U_p t \; ,$$

$$u_g \to 0, \; u_\ell \to 0, \; P_\ell \to P_o, \; \rho_g \to \rho_o, \; T_g \to T_o \text{ as } x \to \infty \; .$$

It is necessary to introduce a dimensionless perturbation parameter ε which is a measure of the shock strength,

$$\varepsilon = \frac{U_p}{a_o},$$

where U_p is the piston velocity and a_o is the equilibrium speed of sound in the undisturbed flow. All the dependent state variables ψ can be expanded in terms of ε as (Van Dyke 1975):

$$\psi = \psi^{(0)} + \varepsilon\psi^{(1)} + \varepsilon^2\psi^{(2)} + - - -,$$

and in particular

$$u_\ell = \varepsilon a_o(u_\ell^{(1)} + \varepsilon u_\ell^{(2)} + \varepsilon^2 u_\ell^{(3)} + - - -), \tag{2.7a}$$

$$u_g = \varepsilon a_o(u_g^{(1)} + \varepsilon u_g^{(2)} + \varepsilon^2 u_g^{(3)} + - - -), \tag{2.7b}$$

$$\beta = \beta_o(1 + \varepsilon \beta^{(1)} + \varepsilon^2 \beta^{(2)} + - - -), \tag{2.7c}$$

$$P_g = P_o(1 + \varepsilon P_g^{(1)} + \varepsilon^2 P_g^{(2)} + - - -), \tag{2.7d}$$

$$T_g = T_o(1 + \varepsilon T_g^{(1)} + \varepsilon^2 T_g^{(2)} + - - -), \tag{2.7e}$$

$$\rho_g = \rho_o(1 + \varepsilon \rho_g^{(1)} + \varepsilon^2 \rho_g^{(2)} + - - -), \tag{2.7f}$$

where ψ_o represents the state variable in the initial stationary state and $\psi^{(n+1)}/\psi^{(n)}$ is of order $O(\varepsilon)$.

The perturbation scheme is illustrated with the continuity equation of the liquid (2.1). After a change of variable from (x,t) to the dimensionless variables (ξ,τ):

$$\xi = \frac{U_p}{r_g a_o} x \quad , \quad \tau = \frac{U_p}{r_g} t,$$

the continuity equation (2.1) becomes

$$a_o \partial_\tau (1-\beta) + \partial_\xi (1-\beta) u_\ell = 0. \tag{2.8}$$

Upon substitution of the perturbation expansions (2.7) in equation (2.8) we obtain to order $O(\varepsilon)$:

$$\beta_o \partial_\tau \beta^{(1)} + (1-\beta_o) \partial_\xi u_\ell^{(1)} = 0. \tag{2.9a}$$

Proceeding in a like manner for equations (2.2) to (2.6) we obtain to order $O(\varepsilon)$ a system of partial differential equations where all dependent perturbation variables are of first order:

$$\partial_\tau(\rho_g^{(1)} + \beta^{(1)}) + \partial_\xi u_g^{(1)} = 0, \tag{2.9b}$$

$$\partial_\tau p_g^{(1)} - \gamma_g \partial_\tau \rho_g^{(1)} + \alpha T_g^{(1)} = 0, \tag{2.9c}$$

$$\partial_\xi p_g^{(1)} + \frac{\Gamma}{2}\,\partial_\tau(u_g^{(1)} - u_\ell^{(1)}) + (u_g^{(1)} - u_\ell^{(1)})\alpha^* = 0, \tag{2.9d}$$

$$(1-\beta_o)\partial_\tau u_\ell^{(1)} + \partial_\xi p_g^{(1)} = 0, \tag{2.9e}$$

$$p_g^{(1)} - \rho_g^{(1)} - T_g^{(1)} = 0, \tag{2.9f}$$

where $\alpha^* = 9\nu_\ell(2U_p r_g)^{-1}$ is the Reynolds number of a typical gas bubble and $\alpha = \gamma_g r_g(U_p\alpha_T)^{-1}$. Here α_T is the thermal relaxation time of a gas bubble defined by $\alpha_T = mc_p(4\pi r_g \sigma_\ell)^{-1}$.

The system of equations (2.9) can be solved by taking the Laplace transform of ψ with respect to the time variable τ:

$$\bar{\psi}(\xi,s) = \int_o^\infty e^{-s\tau}\psi(\xi,\tau)d\tau, \tag{2.10}$$

The operation of multiplying equation (2.9) by the kernel of the Laplace transform and integration with respect to τ over the interval $(0,\infty)$ together with (2.10) and the initial conditions, give set of six coupled auxiliary equations. Decoupling this set of equations for the state variables ψ leads to the algebraic equation:

$$\bar{u}^{(1)}_{\ell,\xi\xi} - s^2 h(s)\bar{u}^{(1)}_\ell = 0, \tag{2.11}$$

where the coefficient h(s) is:

$$h(s) = \frac{(\alpha + s)(1 - \beta_o)(\Gamma s/2 + \alpha^*)\beta_o}{(\gamma_g s + \alpha)(s(\Gamma/2 + \beta_o(1-\beta_o)) + \alpha^*)}.$$

This procedure has reduced the problem to the solution of an ordinary differential equation (2.11), provided $s \neq -\alpha\gamma_g^{-1}$ and $s \neq -a^*\left(\Gamma/2 + \beta_o(1-\beta_o)\right)^{-1}$. The solution of (2.11), for $\xi > 0$ satisfying the boundary conditions is

$$\bar{u}_{\ell}^{(1)}(\xi,s) = \frac{1}{s}\, e^{-s\sqrt{h(s)}\,\xi} . \tag{2.12a}$$

The inversion to $u_{\ell}^{(1)}(\xi,\tau)$ is now accomplished by use of the inversion formula for Laplace transforms and gives

$$u_{\ell}^{(1)}(\xi,\tau) = \frac{1}{2\pi i} \int_{\Sigma} \frac{\exp(s\tau - s\sqrt{h(s)}\,\xi)}{s}\, ds, \tag{2.12b}$$

where Σ is the path of integration, such that Re(s) is constant and is situated to the right of all singularities. An analytic solution to equation (2.12b) can be derived for certain limiting and important cases. For s large, we have the case corresponding to high frequencies and consequently this approximation is valid when the high frequency waves dominate ie., τ small, or near discontinuities in the wave form. For s small, corresponding to τ large, the high frequency waves are attenuated so that lower frequency waves dominate. For s small the expansion for h(s) is

$$h(s) = \sqrt{B} - sA\sqrt{B} + O(s^2),$$

where the dimensionless constants A and B are defined by

$$A = \frac{1}{2}\left(\frac{\gamma_q - 1}{\alpha} + \frac{B^{-2}}{\alpha^*}\right), \quad B = \left(\beta_o(1-\beta_o)\right)^{-1/2} .$$

The constant B represents a dimensionless velocity at which the wave propagates and has a minimum value when $\beta_o = 1/2$. The constant A incorporates the effects of viscosity and thermal conductivity.

3. SOLUTION FOR LARGE TIMES

Using the method of steepest descent the above integral can be evaluated (Van Dyke (1975)) and the solution in non-dimensional form, is given by

$$u_{\ell}^{(1)}(\xi,\tau) = \frac{1}{2}\, \mathrm{erfc}\left(\frac{\xi - B\tau}{2B\sqrt{A\tau}}\right). \tag{3.1}$$

When the above solution is substituted into (2.9) we find

$$u_{\ell}^{(1)} = u_{q}^{(1)}, \quad T_{q}^{(1)} = 0, \tag{3.2}$$

and for the pressure and density:

$$\rho_g^{(1)} = \frac{\beta_o^{-1/2}}{2}(1-\beta_o)\mathrm{erfc}\left(\frac{\xi - B\tau}{2B\sqrt{A\tau}}\right) = p_g^{(1)}. \tag{3.3}$$

It can be seen from (3.1) that this wave travels with velocity Ba_o in an (x,t) coordinate system, and diffuses due to the combined effects of viscosity and thermal conductivity with a characteristic diffusion width given by:

$$\xi - B\tau = 2B\sqrt{A\tau},$$

or

$$x - a_o Bt = 2a_o B\left(\frac{r_g At}{U_p}\right)^{1/2}.$$

The picture of the fluid and gas emerging from the above analysis is that, provided τ is large, the bubbles behave isothermally (since $T_g = T_o$ from (2.7e)) and propagate with the velocity of the fluid ($u_\ell = u_g$). The wave described by (3.1) has a further property that as $A \to 0$, the wave becomes steeper and when it becomes discontinuous then the solution represents a wave for which no dissipative processes are present.

4. SOLUTION FOR SMALL TIMES

An approximate evaluation of the integral (2.12b) corresponding to the high frequency waves (i.e., s large, small time) can be found by expanding s large in h(s) to give:

$$h(s) = a + abs^{-1} + O(1/s^2),$$

where

$$a = \left(\frac{\Gamma}{\gamma_g(\Gamma B^2 + 2)}\right)^{1/2} \text{ and } b = \left(\frac{\alpha(\gamma_g - 1)}{\gamma_g} + \frac{4a\gamma_g\alpha^*}{\Gamma^2}\right).$$

Substitution of h(s) into (2.12a) and integration leads to a liquid dynamic wave whose velocity is given by

$$u_\ell^{(1)}(\xi,\tau) = H(\tau - \xi a)e^{-ab\xi}, \tag{4.1}$$

and moreover

$$u_{\ell}(\xi,\tau) = \varepsilon a_o H(\tau - \xi a) e^{-ab\xi} + O(\varepsilon^2) ,$$

where $H(x)$ is the Heaviside step function defined as $H(x) = 1$ for $x>0$ and $H(x) = 0$ for $x<0$. The above equation indicates a wave which travels at a speed a, and decays exponentially with a decay length $x = r_g a_o (abU_p)^{-1}$, where ℓ_m is the mean free path for transfer of momentum by the gas bubbles. The first order perturbations in pressure, density, temperature and velocity of the gas are respectively:

$$p_g^{(1)} = (1-\beta_o)a^{-1}H(\tau-\xi a)e^{-ab\xi} = \gamma_g^{-1}\rho_g^{(1)} , \tag{4.2}$$

$$T_g^{(1)} = (\gamma_g-1)(1-\beta_o)\gamma_g^{-1}H(\tau-\xi a)e^{-ab\xi}, \tag{4.3}$$

$$u_g^{(1)} = 2[(1-\beta_o) + {}^1\!/_2\,\Gamma]\Gamma H(\tau-\xi a)e^{-ab\xi} .$$

Qualitatively, the gas bubbles move faster than the fluid since $u_\ell^{(1)} < u_g^{(1)}$ i.e., $u_\ell < u_g$. From (4.3) and (2.7e) we find that the temperature of the gas is greater than the fluid (i.e., $T_g > T_o$). The wave for small time, propagates at the speed a, and decays exponentially due to the effects of viscosity and thermal conductivity. The process occurs isentropically.

5. DISCUSSION

A theoretical model has been developed describing a two-phase liquid gas mixture in one dimension. In this model, the Navier-Stokes equations are firstly linearized then decoupled and finally solved by Laplace transform technique. We find that for small times high frequency waves dominate and the gas bubbles in the liquid propagate faster than the fluid. Thermodynamically the bubbles behave isentropically.

In the case where times are large, it is found that the high frequency waves are attenuated so that the lower frequency waves dominate. The gas-bubbles in this case behave isothermally with the shock wave approaching a steady state. For the low frequency case, the width of the transition zone or shock thickness δ, based on the maximum slope:

$$\delta = \frac{U_p}{\left(\frac{\partial u}{\partial \xi}\right)_{max}} ,$$

can be calculated and leads to δ increasing indefinitely (i.e., as $t \to \infty$)

as $\sqrt{t}$. wFor a steady weak shock wave it is well known that $\delta = O(\varepsilon^{-1})$, so as $t \to \infty$, we would expect the linearized solution to break down when $t = O(\varepsilon^{-2})$. That is when t is large compared to ε^{-2}, the linearized solution yields an overthick shock wave. For this case a uniformly valid solution can be found using singular perturbation methods, and will be discussed in a following paper.

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